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NUMBER FUN WITH A CALENDAR

AN ALARSRI PUBLICATION

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Number Fun with A Calendar

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1988 Calendar Sheet

JAN

S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

FEB

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29					

MAR

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

APR

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

NUMBER FUN
WITH
CALENDAR

(Age group 11-14)

P.K. SRINIVASAN

July '88

MAY

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

JUN

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

JUL

S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

AUG

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

SEP

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

OCT

S	M	T	W	T	F	S
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

NOV

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

DEC

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

ENJOYMENT OF MATHEMATICS IS THE BIRTHRIGHT OF EVERY BOY AND GIRL. THIS BOOK SHOWS HOW EVERY BOY AND GIRL CAN EXERCISE IT AND DISCOVER HIS/HER MATHEMATICAL POTENTIAL IN A STIMULATING ENVIRONMENT

— *Parents and Teachers*

PREFACE

The young and the old get into a hunting spree for calendars about the beginning of each new year. One who succeeds in getting a large number of them feels proud that he can excite the envy of his near and dear.

Calendars are hung prominently in homes and schools, shops and offices and so there cannot be anyone who is not familiar with the arrangement of numbers in seven columns or rows. But the fascinating patterns that emerge out of this arrangement are scarcely known.

This book attempts to acquaint children with some of the wonderful patterns in a calendar arrangement of numbers and provide them with a new source of mathematical enjoyment. Once children are exposed to look for patterns in a common place material like calendar, they are likely to develop a taste for mathematics. They get highly excited and hence motivated and that is the first great step in securing their willing involvement in mathematical learning.

In these days when kindergarten is considered too late, this little book will help in catching children young for mathematics. Once children acquire basic skills of addition and multiplication, they can be considered to be ready for number fun with a calendar sheet.

A topic that has appeal for different age groups would be most suitable and the calendar arrangement of numbers belongs to this category. It lends itself to understanding of concepts at various levels. Personalised presenation has an instant appeal for children as it invites intuitive thinking with an arresting directness and so this mode is adopted in this children's book. In the absence of extra reading, the

interested in our country become often indifferent. My experience has shown that conversational mode makes the interested involved and the involved talented. Mathematically talented children are in great demand in this breathtaking age of chips.

Incidentally the material is so presented that video taping and televisability can be easily secured. The scope of knowing enjoyable mathematics through number arrangement on a calendar in a recreational spirit is really remarkable, as anyone with a mathematical flair could easily discover for himself/ herself.

P.K.SRINIVASAN

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"Any idea or problem or body of knowledge can be presented in a form simple enough so that any particular learner can understand it in a recognizable form" - *James S. Bruner*

JOIN US

I am Prasad, studying in class IX. I have a younger brother Ravi, studying in class VII and a little sister, Susheel in class V.

Our father does textile business and he loves to spend weekends with us. He sees mathematics in everything around us and because of him, we know how to play mathematical games with a tumbler set, a belt, a strip of paper, a ruled sheet, railway tickets etc. All the games have whetted our liking for mathematics.

What he does is to accept anything brought by us and devise to our surprise games involving it. Thanks to him, we have discovered that there cannot be any game without some mathematics in it.

May 1 this year, 1988, happened to be a Sunday and he was to have a spell of holidays for a month. We asked him what fun we could get from the arrangement of numbers on the calendar sheet for May. That was all. All the weekends of May, we had an exciting time just with the calendar.

He would just throw us a hint or two and wonderful patterns continued to surface to our great delight.

We place before you the story of our discoveries on a calendar sheet and we invite you to join us, enjoy fun and share it with others.

To start with, he asked us to bring our slates. On Susheel's slate he wrote the following arrangement:

				1	2	3	
4	5	6	7	8	9	10	11
12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27
28	29	30	31				

and asked us if there could be an arrangement of this kind on any month's calendar. At first we wanted to campare it with arrangements in the sheets of the calendar. But he said we should not do it but answer his question by mere inspection of the arrangement.

The first turn went to Susheel. She said she was lost. Ravi said that there could only be seven columns to go with seven days and so the arrangement dad had written was not a calendar arrangement. I said that the numbers in any column in a calendar increased by seven but in dad's arrangement the numbers in any column increased by eight and so the dad's arrangement was not a calendar arrangement. Susheel warmed up and said that on a calendar skip counting in columns was in sevens and not in eights, as a week has only seven days.

Thus the story of our explorations on a calendar sheet began.

INSTANT MEAN AND INSTANT SUM

D: Here is May '88 calendar

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Ravi, show me any row. I shall give you instantly the mean of the numbers in it.

R: Second row.

D: 11.

P: I know it. It is the middle number.

D: Be careful. We must know when a sequence of numbers has its middle number as its mean. Let us first see what kinds of sequence we have on a calendar sheet.

P: We have sequences row wise, column wise and diagonal wise. Each sequence is an arithmetic progression.
Row wise, numbers in each sequence increase by 1.

For example 1 + 1 is 2, 2 + 1 is 3 etc.

D: Column wise?

R: Numbers in each sequence increase by 7

For example 1 + 7 is 8, 8 + 7 is 15 etc.

D: Diagonal wise?

P: There are two diagonal wise sequences.
For the left top to right bottom diagonal wise sequence the increase is 8.

For example 1 + 8 is 9, 9 + 8 is 17 etc.

For the right top to left bottom diagonal wise sequence it is 6.

For example 4 + 6 is 10, 10 + 6 is 16 etc.

D: We can also call such sequences additive sequences as the same number is added to a term to fix the next term in such a sequence.

R: Mean of numbers in the first column is 15. Am I right?

D: Yes, you are! Now I say 17 is the mean of two sequences. Can you point them out?

R: 3, 10, 17, 24 and 31.

5, 11, 17, 23 and 29.

S: Why not 15, 16, 17, 18, 19, 20 and 21?

R: 17 is not the middle number in this sequence. Daddy, I have a question. Do you mean to say that if we add the numbers in a sequence and divide the sum by their number the mean got as the quotient will be the middle number of the sequence?

D: Try and see.

R: $$\frac{(1 + 2 + 3 + 4 + 5 + 6 + 7)}{7} = \frac{28}{7} = 4$$

$$\frac{(1 + 8 + 15 + 22 + 29)}{5} = \frac{75}{5} = 15$$

Yes, it is correct.

D: I can also give the instant sum, if you show me a sequence.

S: Second row.

P: 77, correct?

R: Oh, I see, the mean of 7 numbers is 11 and so the sum of 7 numbers is 11 x 7 or 77.
Alright. What about numbers in the 4th column?

D: There is no middle number, of course. But I can give the instant sum. It is 58.

S: 4 + 11 + 18 + 25 = 58. Yes, it is correct. How come?

R: How do you get it, daddy?

D: Well, the number of terms is 4. That is, the number of terms is even. In that case add the end terms

i.e., 4 + 25 = 29 here.

Multiply the sum by half the number of terms

i.e., 1/2 x 4 here.
You get the sum of the terms
29 x 2 = 58.

S: When there is an odd number of terms there is a middle number. When there is an even number of terms, there is no middle number.

P: When an odd number of terms are in an additive sequence, the middle number becomes their mean. When that mean is multiplied by the number of

terms, the sum of the terms is got.
When an even number of terms are in an additive sequence .....

R: There is no middle number, but you can get their sum by adding the end numbers and multiplying the answer by half the even number.

D: They need not be end numbers. Take the numbers immediately after and immediately before the end numbers, add them and see.

R: 4, 11, 18, 25. The number after 4 in this sequence is 11 and the number before 25 is 18. Add 11 and 18. 29 is got. It is the same as
4 + 25 = 29.

D: That is good. So instead of end numbers, we can add equally placed numbers.

P: So, daddy, equally placed terms form pairs
e.g. (4, 25), (11, 18).
Each pair gives the same sum. The number of pairs is half the number of terms
e.g. 2 = 1/2 x 4.
So multiply a pair sum by the number of pairs and the sum of the terms is got.

D: You have put it beautifully. Have I told you a story about Gauss, the great mathematician?

S: No, tell us the story, daddy.

D: Well, listen. The boy Gauss was in primary three. His teacher once gave an exercise to keep the class busy the whole period. It was to write numbers from 1 to 100 and then add them up. The boy who did it first was asked to place his slate on the teacher's table. Gauss who had started doing many things in mathematics by himself was excited, as he had discovered how to do it instantly. He wrote in his slate 5050 as the sum of the numbers 1 to 100, got up and was the first to place his slate on the table.

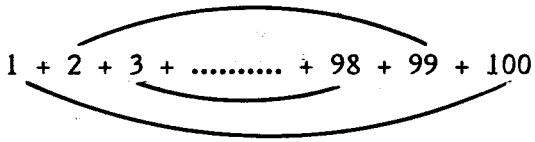
R: Did he write down the numbers 1 to 100?

D: No.

S: The teacher should have been angry with him.

D: Yes, he was angry. But his surprise overcame his anger. He asked Gauss how he knew the answer.

R: Oh, he should have formed pairs of equally placed numbers.



Each pair sum is 101. There are 1/2 x 100, that is 50 pairs. So 101 x 50 = 5050.

P: This he did all by himself?

D: Yes and thus showed his mathematical potential

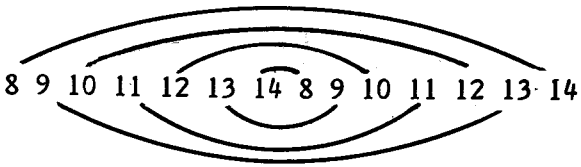
S: Oh, God! that is why he became a great mathematician.

P: Why should there be two different methods? Can't we have a single formula?

D: That is a good question. To get a single formula, we should look for another pattern. Take the sequence of numbers in row 2 for instance and write beside it the same sequence.

R: 8 9 10 11 12 13 14 8 9 10 11 12 13 14

D: Form pairs of equally placed numbers.

R: 

D: What is the sum of each pair?

S: 22.

D: How many pairs are there?

R: 7.

D: What is the sum of the 7 pairs?

S: 7 x 22 = 154. But we should get 77.

P: Yes, we have taken twice the sum of terms in the sequence. So we should find 1/2 the sum got.

S: I see. 1/2 of 154 is 77. It is correct.

D: 8 is the first term and 14 is the last term of the sequence chosen.
8 + 14 is the pair sum. There are 7 terms. The sum of all the terms taken twice is 7 x (8 + 14). But we want the sum of all the terms when taken once. So the sum is 1/2 x 7 x (8 + 14). To give the formula we shall denote the first term by a, the last term by l and the number of terms by n. What will be the formula?

R: 1/2 x n x (a + l).
What about sequences with even number of terms?

P: The same formula holds good. Take 4, 11, 18, 25. Let us do the same thing as before

4 11 18 25 4 11 18 25

R: The sum of each pair of equally placed numbers is 29. There are 4 pairs. The sum of four pairs is 4 x 29. We need only half the sum. So it is 1/2 x 4 x 29.
This is really nice. But you gave two methods, one for a sequence with odd number of terms, another for a sequence with even number of terms. How is it?

D: It is simply two interpretations of the same formula. Look. Take the formula 1/2 x n x (a + l). Can't we say it is the same as

$$n \times \frac{(a + l)}{2} \quad \text{or} \quad 1/2 \times n \times (a + l)$$

P: Oh! I see. $\frac{(a + l)}{2}$ is the mean. In our sequence

it is the middle term and n the number of terms. This is your first method to find the sum when there are an odd number of terms.
(a + l) is the sum of end terms and 1/2 x n is the number of pairs. This is your second method to find the sum when there are an even number of terms.

3

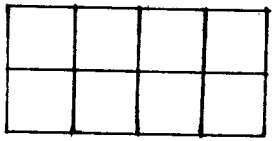
MAGIC RECTANGLES

D: When you find something repeating, what do you call it?

S: Pattern.

D: With equal square bits, how do you build a rectangle?

S: I shall take my square bits. Here are some.
Let me build:



This is a rectangle.

D: That is good. In a rectangular arrangement, we have numbers in more than one row and each row has the same number of numbers. You can call it, if you like a rectangular matrix.

S: Let me show one in the calendar

2 3 4 5

9 10 11 12

D: This is a rectangular matrix of order 2 by 4, that is 2 rows and 4 columns.
Now can you find the magic in it?

S: I get it

2 + 12 = 14

3 + 11 = 14

4 + 10 = 14

5 + 9 = 14

so it is a magic rectangle!

R: Here is my choice.

1 2 3

8 9 10

15 16 17

22 23 24

29 30 31

It is of 5 by 3 order. Right?

D: Good. What is the magic in it?

1 + 10 = 11 8 + 17 = 25

3 + 8 = 11 9 + 16 = 25

2 + 9 = 11 10 + 15 = 25

15 + 24 = 39 22 + 31 = 53

16 + 23 = 39 23 + 30 = 53

17 + 22 = 39 24 + 29 = 53 and so on.

P: 11, 25, 39, 53 form an additive sequence.

R: Why not I take a square arrangement like

5 6 7

12 13 14

19 20 21

and look for magic in it?

D: We shall consider square matrices in our next sitting.

S: I want to take a big rectangular matrix. Here is one.

2 3 4 5 6 7

9 10 11 12 13 14

16 17 18 19 20 21

23 24 25 26 27 28

D: What is its order?

S: It is 4 rows by 6 columns.

D: You mean 4 by 6. O.K. Telf us all the magic you find in it.

S: 2 + 28 = 30 9 + 21 = 30

3 + 27 = 30 10 + 20 = 30

4 + 26 = 30 11 + 19 = 30

5 + 25 = 30 12 + 18 = 30

6 + 24 = 30 13 + 17 = 30

7 + 23 = 30 14 + 16 = 30

P: If we take sub-rectangular matrices from this, we can find more magic.

D: That is fine. Let us not bother about them.

INCOMPLETE MAGIC SQUARES 4

P: So we are going to take square arrangements and find the magic in them.

D: Well, let Susheel take the first turn.

R: What order, daddy?

D: Let us start with 2 by 2 or simply 2nd order

S: 1 2 3 4

8 9 10 11 Am I right?

R: Why not?

10 11 20 21

and

17 18 27 28

D: That is also correct. Now what is the magic in these?

S: 1 + 9 = 10 3 + 11 = 14 10 + 18 = 28

2 + 8 = 10 4 + 10 = 14 11 + 17 = 28

20 + 28 = 48

21 + 27 = 48

P: Diagonal sums alone are equal. What happens when we take a 3rd order square matrix?

R: Here is one:

1 2 3

8 9 10

15 16 17

Diagonal sums are

1 + 9 + 17 = 27

3 + 9 + 15 = 27

D: What about the middle row?

S: 8 + 9 + 10 = 27

The middle row sum is the same. Let me check up the middle column sum now.

2 + 9 + 16 = 27

Oh! This sum is also the same. This 3rd order square matrix is more interesting. Why can't we have the same sums in the first row and the third row?

Why can't we have the sums in the first column and the third column the same?

D: Then it would become a complete magic square.

R: So this is an incomplete magic square. Daddy, I think I can discover something here. In an incomplete magic square of this kind, the magic sum can be got by multiplying the central figure by 3, the order of the square.

P: Can it be true always? Let me check:

12 13 14

19 20 21

26 27 28

The central figure is 20. The order of the square is 3. So the magic sum is 20 x 3 = 60.
The diagonal sums are

12 + 20 + 28 = 60 and 14 + 20 + 26 = 60

S: The middle row sum is

19 + 20 + 21 = 60

and the middle column sum is

13 + 20 + 27 = 60

P: This is wonderful. We can also pick out a 4th order square matrix.

D: Yes, note that you can go only up to 4th order in a calendar. If you want higher order squares, you will have to extend the numbers beyond 31. Even then there is a limit. Can you tell us what it would be?

R: How can we say it, unless we know where you stop?

S: There can be only 7 columns; so there can only be 7th order magic square.

P: That is splendid, daddy?

D: I agree. Well, take a 4th order square matrix and find out all the magic you can get out of it.

R: 4 5 6 7

11 12 13 14

18 19 20 21

25 26 27 28

This is a 4th order square matrix.

S: 4 + 12 + 20 + 28 = 64

7 + 13 + 19 + 25 = 64

P: 5 + 6 + 26 + 27 = 64

11 + 14 + 18 + 21 = 64

12 + 13 + 19 + 20 = 64

4 + 7 + 25 + 28 = 64

D: Do you know what you are doing?
You are just taking complementary pairs for 32 twice.

P: It is true. In a magic rectangle we take complementary pairs once.

R: Well, daddy, can't we rearrange the numbers to get a complete magic square?

D: That is a good question. We shall take it up later. If you are interested to find it earlier by trial and error, you can do so. You have missed one point. In a 3rd order square, you find the magic sum by multiplying the central figure by 3. How do you find the magic sum in a 4th order square in a similar manner?

P: There is no central number. So I find the sum of numbers in opposite corners and multiply it by 2 to get the magic sum.
Now take this 4th order square.

2	3	4	5
9	10	11	12
16	17	18	19
23	24	25	26

R: The magic sum

(2 + 26) x 2 = 56

S: I can also find it by adding 5 and 23 and multiplying the sum got, that is, 28 by 2.

D: There are also other similar ways of finding the sum.

R: Suppose we want to find the grand sum of all the numbers in a calendar square?

D: Take any 3rd order square matrix. Give me the top left corner number. I shall give the grand sum.

S: I have chosen a 3rd order square matrix. The top left corner number is 12.

D: The grand sum is 180.

S: Let me check:

12	13	14	
19	20	21	
26	27	28	
57	60	63	180

Correct.

R: I get it. Multiply the central number by 9.

S: But I did not give the central number.

R: Then add 8 to the top left corner number. Right?

P: That is fantastic. Why the top left corner number alone? If you give the top right corner number, we will have to add 6 to get the central number and then multiply the central number by 9 to find the grand sum.

D: Oh! you have discovered the secret of the game! Good.

P: I can go further now. Take a 4th order square and give me the top left corner number. I will give the grand sum.

S: 4 is the left top corner number of my 4th order square.

P: 256 is the grand sum.

S: Let me check and see

4	5	6	7	
11	12	13	14	
18	19	20	21	
25	26	27	28	
58	62	66	70	256

R: Wait. I will find out the secret. First of all, we must get the sum of the diagonal along 4. To get the sum, we must know 28. For that add 3 times 8 to 4, as you skip 3 places in the sequence. Then add 4 and 28, get 32 and multiply 32 by 2. You get 64. Multiply 64 by 4 and you get the grand sum.

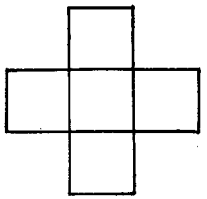
P: That is great and wonderful.

CROSS PUZZLES

D: The calendar arrangement of numbers shows the way of setting some cross puzzles and solving them.

S: Give us a cross puzzle. We shall solve it.

D: Here is one. We can call it 3rd order cross puzzle.



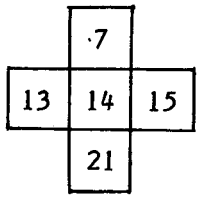
Fill up the cells in the 3rd order cross such that the vertical sum is the same as the horizontal sum.

S: There are so many solutions. I shall pick out two.

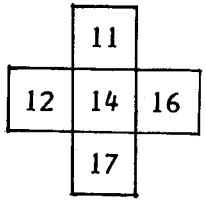
	3			13	
9	10	11	19	20	21
	17			27	

D: Good. Suppose I give the cross sum first. It is 42. Can you fix the 3rd order cross from it?

R: First we should find the number in the central cell. It is 42/3 = 14. Then subtracting 7 from and adding 7 to 14, vertical cells are fixed. Subtracting 1 from and adding 1 to 14, horizontal cells are fixed. The solution will be

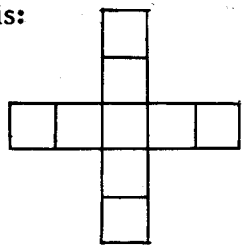


P: Only if we want a calendar type solution, we have to use 7 and 1. There can be non-calendar type solutions. Once we fix the figure for the central cell by dividing the given sum by three, we can choose a convenient number, say, 3, subtract it from and add it to the central figure 14 and fix the vertical cells with 11 and 17. Then choose some other convenient number, say 2, subtract it from and add it to the central figure 14 and fix the horizontal cells with 12 and 16.



R: Can't we have a cross puzzle of higher order?

P: Next to 3rd order cross puzzle, we can have 5th order cross puzzle like this:

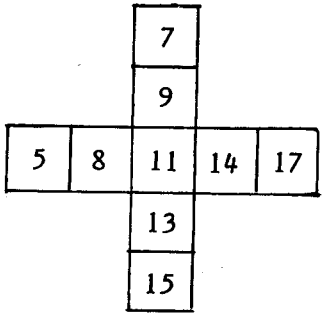


S: I cannot solve this with the calendar arrangement.

R: Why? Give me cross sum or central figure I shall solve it.

P: O.K. The cross sum is 55.

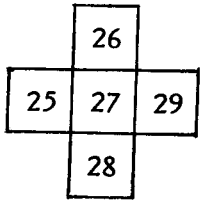
R: The central figure is 11. Subtracting 2 successively twice, we get 9 and 7. Adding 2 successively twice to the central figure, we get 13 and 15. These can go respectively into two vertical cells above and the two vertical cells below the central cell. Subtracting 3 successively twice from the central figure 11, we get 8, 5 and put them in the two horizontal cells to the left. Adding 3 successively twice to the central figure, we get 14, 17. These can go into the two horizontal cells to the right. So the solution is:



R: Another way strikes me now. Why not we fix two five term additive sequences with 11 as the common middle term.

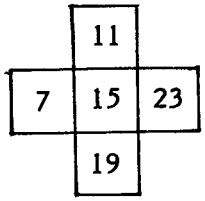
D: That would be pretty! Well, we shall put the puzzle in another way. There are five consecutive numbers, say, 25,26,27,28 and 29. How will you use these to solve a 3rd order cross puzzle?

R: The middle figure should be the central cell figure. Well, here is the cross:



P: You are really smart! I see the point. When we choose 2 and 1 as figures for doing subtraction and addition we get this nice arrangement. I get another idea. The five numbers need not be consecutive. They can be in an additive sequence. Suppose we take 7, 11, 15, 19, 23 we can fix the cross.

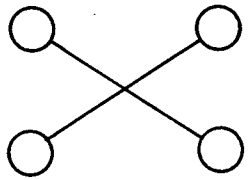
S: Let me try:



I have got it.

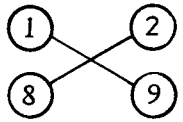
STAR PUZZLES 6

D: We have seen cross puzzles. We can now think of star puzzles.



This is a star puzzle of order 2. How will you solve it to get the two diagonal sums the same.

S: It is easy if we use the calendar.



Here is one solution with star sum 10.

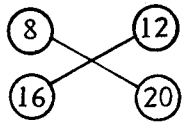
R: If you give me star sum, I shall fix the star.

D: O.K. The star sum is 28.

R: 28 should be expressed as sum of two numbers in two ways

$$28 = 20 + 8; \quad 28 = 12 + 16$$

Here is the solution:



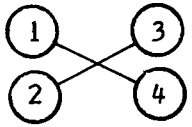
D: 20 and 8 form a complementary pair for 28; so are 12 and 16. You can use this language if you like.

S: But this is not a calendar type solution. So if we

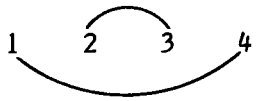
want a calendar type solution, we can give these:

R: Can't we use four consecutive numbers and fix this star?

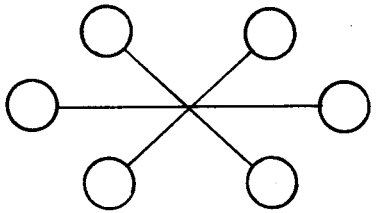
S: I get it:



I have to take equally placed number pairs.



P: A good shot! Suppose we consider a star of 3rd order? How do we fill the cells?



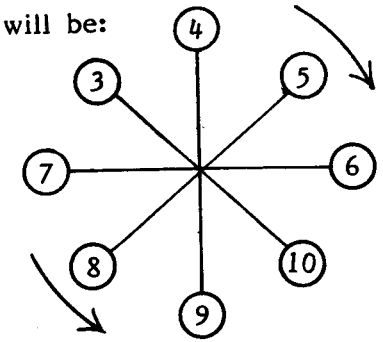
R: It will be simply the numbers in a 3rd order incomplete magic square with the central number left out.

P: Daddy, shall we say the order of a star puzzle is the number of complementary pairs making the star?

D: Oh, yes. You can say so. Look, I give you 8 consecutive numbers, say, 3, 4, 5, 6, 7, 8, 9, 10. Can you use them to fix the star of 4th order.

R: The pairs are:
(3,10), (4,9), (5,8), (6,7).

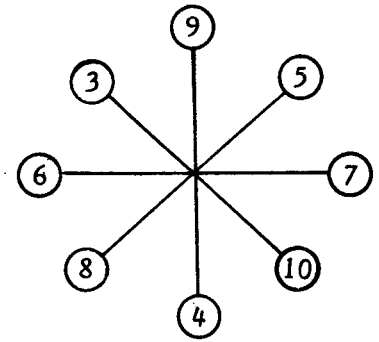
The star will be:



P: 3,4,5,6 are in four consecutive cells in clockwise order. 7,8,9,10 are in four consecutive cells in anticlockwise order.

D: Should they be like that?

R: I don't think so. It can also be like this:



P: Good! But I like this cyclic and anticyclic orders appearing in the star. As before, we can also have 8 numbers in additive sequence to fix a 4th order star.

7 COMPLETE MAGIC SQUARES

D: Ravi's question now. Can we rearrange the numbers in a square arrangement to get a complete magic square?

S: You mean all row sums, all column sums and the two diagonal sums can be made equal?

D: Yes, I do. Let us try.

R: That will be terrific!

D: Let us start with a 2nd order square of numbers on a calendar.

S: Here is a 2nd order square:

1	2
8	9

However much I try, I cannot change it into a complete magic square.

R: She is correct. It is impossible.

P: Let us now take a 3rd order square of numbers and see.

1	2	3
8	9	10
15	16	17

S: Let me check up.

8 + 9 + 10 = 27

2 + 9 + 16 = 27

17 + 9 + 1 = 27

3 + 9 + 15 = 27

R: You are very fast!

S: Not at all! Are these not additive sequences seen in the calendar arrangement? So each sum is simply
3 x 9 or 27.

R: The magic sum should be 9 x 3 = 27
How to rearrange the numbers to make the square a complete magic square?
Daddy, give us a hint.

D: Well, you can do it by trial and error; it will be time consuming. There is a hidden structure. You learn to see it first. Now how will you get 8, 9 and 10 from 1,2 and 3?

S: By adding 7 to each of 1,2 and 3 we get 8,9 and 10.

R: By adding 14 to each of 1,2 and 3 we get the last row 15,16 and 17.

D: Good. How do we get 1,2 and 3 from 1,2 and 3?

S: By adding 0 to each of 1,2 and 3.

D: What we have now to do is to build two squares, called auxiliary Latin squares, one with 1,2 and 3 and the other with 0,7 and 14.

P: What is a Latin square, daddy?

D: I was about to explain. In a Latin square, the same numbers are used in each row or column, but a number is not repeated in the same row or

column.

R: What about a diagonal?

D: We ignore the diagonal. A number can get repeated more than once along a diagonal.

S: Should the row sums, the column sums and the diagonal sums be all equal?

D: No, it is enough if the row sums and column sums are equal.

R: Then even a 2nd order Latin square can be built.

e.g. 1 2

2 1

S: Let me try to build a Latin square with 1,2 and 3. 2 should be in the central cell.

1		
	2	
		3

1	3	2
3	2	1
2	1	3

Yes, I have built it.

R: I have built the Latin square with 0, 7 and 14.

0	14	7
14	7	0
7	0	14

S: His is just like mine, daddy,

P: Yes, the replacements can be shown thus:

1 $\longrightarrow$ 0

2 $\longrightarrow$ 7

3 $\longrightarrow$ 14

D: Good! You mean by your arrow 'is replaced by'. O.K.? Now build a square matrix by adding the two auxiliary Latin squares, cell to corresponding cell.

S: Let me do it:

1+0	3+14	2+7
3+14	2+7	1+0
2+7	1+0	3+14

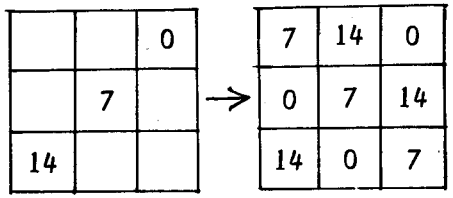
This is what we get!

1	17	9
17	9	1
9	1	17

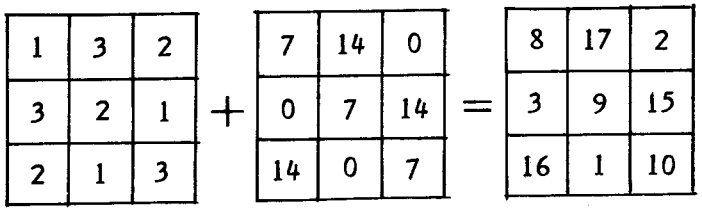
R: Another Latin square, daddy!

D: Suppose you build the Latin square with 0, 7, and 14 in a different pattern and then add ...

P: Let me try:
The order of 1, 2, 3 was from the left top to the right bottom. I will set 0, 7, 14 to be along the diagonal from the right top to the left bottom.



R: Let me add, cell to cell, and get the composite square.



This is not a Latin square. Is this a magic square?

R: Yes, I get it. So this also behaves like a 3rd order square of numbers on a calendar sheet. But is it incomplete or complete?

S: Let me check the other row sums and the other column sums.

8 + 17 + 2 = 27

16 + 1 + 10 = 27

8 + 3 + 16 = 27

2 + 15 + 10 = 27

Oh! This is a perfect magic square!

D: Not only that. Some more magical properties have emerged.

S: What is that, Daddy

D: Try writing the square of the number in each cell.

R: Here it is, right?

64	289	4
9	81	225
256	1	100

P: Now we should check up if this is a magic square? I have found out that it is not a perfect magic square, but an imperfect square of new variety.

64	289	4	357
9	81	225	
256	1	100	357
329		329	

D: What do you mean?

P: Instead of mid row sum, mid column sum, the two diagonal sums being equal, we get here the end row sums and the end column sums alone equal.

D: Don't you see that this solves incidentally the puzzle of finding numbers that can be expressed as a sum of three squares in two different ways!

R: We should look at it that way also. Marvellous indeed!

P: What about incomplete magic square of the 4th order? Can we rearrange and make it into a complete magic square?

D: Oh, yes. You can do it.

P: Daddy, watch us. Let us take

1	2	3	4
8	9	10	11
15	16	17	18
22	23	24	25

How do you call these two sets of numbers we derive. The two sets for this square being made up of 1, 2, 3 and 4 and 0, 7, 14 and 21.

D: Well, 1, 2, 3 and 4 are called *basic numbers* and the others *root numbers*.

P: We shall build the Latin square with basic numbers first. Only after building it, we can build the one with root numbers.

S: There is no central cell and there is no middle number. So how to do it?

R: Well, let me try:

1	2	3	4
2	3	4	1
3	4	1	2
4	1	2	3

P: Good! Let me build the other Latin square with root numbes 0, 7, 14 and 21. 4's appear along R-L diagonal. So 21's should be along L-R diagonal:

21			
	21		
		21	
			21

→

21	0	7	14
14	21	0	7
7	14	21	0
0	7	14	21

1, 2, 3 and 4 are in the top row. So 0, 7 , 14, 21 should be in the bottom row. I have got it. Let us add the two Latin squares, cell to cell.

R: Let me do it.

1+21	0+2	3+7	4+14
2+14	3+21	4+0	7+1
3+7	14+4	1+21	2+0
4+0	1+7	2+14	3+21

=

22	2	10	18
16	24	4	8
10	18	22	2
4	8	16	24

This is too bad!

S: Yes, 24 is repeated, 8 is repeated, 18 is repeated and 10 is repeated. This is no good at all! The diagonal sums are different. So this is not even an incomplete magic square we get from a calendar.

P: Oh! we don't get even a Latin square!
Daddy, we are lost.

D: There are different ways of building a Latin square. Let me build for you an appropriate Latin square with the basic numbers 1, 2, 3 and 4:

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

P: Oh! this is also a Latin sqaure. Alright. Suppose I build the other Latin square with the replacement scheme:

1 → 0

2 → 7

3 → 14

4 → 21

and then add the Latin squares, cell to cell, will I make it?

R: Let us try and see:

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

+

0	7	14	21

P: Stop! We must do the second Latin Square in a different pattern.

R: O.K. Let us change the pattern of the first square matrix to get the second square matrix in a different pattern.
You mean you will continue to use the same scheme of replacement.

R: Yes.

1	3	2	4
2	4	1	3
3	1	4	2
4	2	3	1

→

0	14	7	21
7	21	0	14
14	0	21	7
21	7	14	0

P: Good! Go ahead.

R: Now we should add

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

and

0	14	7	21
7	21	0	14
14	0	21	7
21	7	14	0

cell to cell.

S: Let me do it.

1	16	10	25
10	25	1	16
16	1	25	10
25	10	16	1

P: Oh! We are only getting a Latin square with 10, 16 and 25.

D: Look! You will have to study the formation of my Latin square with the basic numbers a little more deeply.

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

P: Oh! I find it now. See if I am right? Splitting it into four squares of 2nd order, we get:

1	2	3	4	2	1	4	3
3	4	1	2	4	3	2	1

Each has diagonal sums the same number 5. The first and third, the second and the fourth are mirror reflections of each other. The third is below the first and the fourth below the second.

O: Very good! So if we want to build the second Latin square with the root numbers 0, 7, 14 and 21 on a different pattern, how should we go about it?

R: Diagonal sums should not be the same. Row sums and column sums should be the same. Right?

D: Note the use of equal placement in choosing numbers from 1, 2, 3 and 4 to get equal diagonal sums.

P: So we should consider. (0, 21) and (7, 14)

R: Shall we build the four smaller squares thus?

0	21	7	14	21	0	14	7
21	0	14	7	0	21	7	14

D: Good! How shall we compose them to get the 4th order Latin square? Note that a smaller square and its reflection get fixed one below the other in our first Latin square.

P: So we should here put them diagonally:

0	21		
21	0		
		21	0
		0	21

0	21	14	7
21	0	7	14
7	14	21	0
14	7	0	21

R: Let me do the addition:

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

+

0	21	14	7
21	0	7	14
7	14	21	0
14	7	0	21

=

1	23	17	11
24	4	8	16
9	15	25	3
18	10	2	22

P: Daddy, we have hit it!

S: Let me test and see:

$$
\begin{aligned}
1 + 4 + 25 + 22 &= 52,\\
11 + 8 + 15 + 18 &= 52,\\
1 + 23 + 17 + 11 &= 52,\\
24 + 4 + 8 + 16 &= 52,\\
9 + 15 + 25 + 3 &= 52,\\
18 + 10 + 2 + 22 &= 52,\\
1 + 24 + 9 + 18 &= 52,\\
23 + 4 + 15 + 10 &= 52,\\
17 + 8 + 25 + 2 &= 52,\\
11 + 10 + 3 + 22 &= 52,
\end{aligned}
$$

Yes. It is a complete magic square indeed!

P: Daddy, I have a question. In a calendar arrangement, the root numbers are 0, 7 and 14 or 0, 7, 14 and 21. They are simply the consecutive multiples of 7 starting from 0. Can't we build a magic square with consecutive multiples of any other number?

R: You mean root numbers like 0, 6, 12, 18 or 0, 4, 8, 12 etc?

P: Yes, that is it.

D: Why not? Try and see.

S: I will use 1, 2, 3 and 0, 4, 8 to build a 3rd order complete magic square.

R: I will use 1, 2, 3, 4 and 0, 6, 12, 18 to build a 4th order complete magic square.

P: Go ahead.

S:

1	3	2
3	2	1
2	1	3

+

4	8	0
0	4	8
8	0	4

=

5	11	2
3	6	9
10	1	7

S: I get it.

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

+

0	18	12	6
18	0	6	12
6	12	18	0
12	6	0	18

=

1	20	15	10
21	4	7	14
8	13	22	3
16	9	2	19

I get it too!

D: There is a warning. The second of the root numbers should not be less than the last of the basic numbers.

P: I get the point. Otherwise even the incomplete magic square will be imperfect. Taking the second of the root numbers to be less than the last of the basic numbers in 3rd order square making, we get

1 2 3

3 4 5

5 6 7

R: I see a beautiful arrangement coming up when we take the second of the root numbers to be equal to the last of the basic numbers. Let me take the 4th order square matrix:

1 2 3 4

5 6 7 8

9 10 11 12

13 14 15 16

P: Daddy, I know how to change this into a complete magic square by interchanging equally placed

figures along each diagonal:

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

Can't we use this technique in changing an incomplete magic square from a calendar into a complete magic square?

D: Try and see.

R:

1	2	3	4
8	9	10	11
15	16	17	18
22	23	24	25

This is an incomplete magic square. Interchanging 1 and 25, 9 and 17, 4 and 22, 10 and 16, we get:

25	2	3	22	52
8	17	16	11	52
15	10	9	18	52
4	23	24	1	52
52	52	52	52	52

Oh! Yes. I get it.

P: Shall we check up the magic square having squares of entries in it?

D: I was about to suggest it. Go ahead

R: I am already at it

625	4	9	484	1122
64	289	256	121	730
225	100	81	324	730
16	529	576	1	1122
856	930	922	922	930	996

S: Daddy, this is more magical than 3 x 3 squared magic!

P: Oh, we get numbers that can be expressed as sum of four squares in two different ways.

R: Daddy, what would happen if we take cubes of entries?

P: I am already at it.

S: I shall fix the cubes with this pocket calculator

15625	8	27	10648	26308
512	4913	4096	1331	10852
3375	1000	729	5832	10936
64	12167	13824	1	26056
15808	19576	18088	18676	17812	21268

S: Shocking, Daddy. A big flop.

D: Don't be hasty. It is a challenge for exploration.

R: I have an idea. Why should we build the magic square this way? Is there no other way?

D: That is a good question. Try interchanging 2

24, ...

S: I see, 3 and 23, 8 and 18, 15 and 11

R: Here is it

1	24	23	4
18	9	10	15
11	16	17	8
22	3	2	25

S: Does it turn out to be a magic square?

R: It should be

1	24	23	4	52
18	9	10	15	52
11	16	17	8	52
22	3	2	25	52
52	52	52	52	52

Yes, I am right

D: I have a suggestion. Interchange mid columns and check

R: We have only to check diagonal sums

1	23	24	4
18	10	9	15
11	17	16	8
22	2	3	25

52 52

Yes, we get it.

P: You want us to try the square with cubes of entries. Don't you, Daddy?

D: Go ahead

P: We won't be any better.

D: You are right. But a new situation emerges to get our treasure.

R: What is it, Daddy!

D: Form a mid point square

R: you mean this?

1	23	24	4
18	10	9	15
11	17	16	8
22	2	3	25

D: Very good, you have got it

S: What do you want us to do, now?

P: May be, we should add the numbers on the opposite sides of the mid square.

D: That is shrewd indeed! Go ahead

P: Let me check up their square sums

R: Let me check up their cube sums

P: Alright, let us hurry up

S: $18^2 + 23^2 + 3^2 + 8^2 = 926$

$24^2 + 15^2 + 11^2 + 2^2 = 926$

R: $18^3 + 23^3 + 3^3 + 8^3 = 18538$

$24^3 + 15^3 + 11^3 + 2^3 = 18538$

D: So you see, what a treasure hunt it is!

Check up if this kind of magical property can be had with calendar arrangement of numbers taken in a 4 x 4 square matrix.

S: Daddy, I know how to change

1 2 3

4 5 6

7 8 9

into a third order magic square.
1 5 9 and 3 5 7 are taken to middle column cells and middle row cells. 2 5 8 and 4 5 6 are taken to diagonal cells in the reverse order.

	1	
3	5	7
	9	

8	1	6
3	5	7
4	9	2

Can I change a 3rd order incomplete magic square from a calendar into a complete magic square by this method?
Let me first try and see

1 2 3

8 9 10

15 16 17

1 9 17 and 3 9 15 should go into middle column and middle row.

2 9 16 and 8 9 10 should go into diagonals in the reverse order.

So I get:

16	1	10	27
3	9	15	27
8	17	2	27
27	27	27	27 27

Oh! I see, I am getting it.

8

COLUMN SUM GAME

S: Is there no number game like 'Think of a Number' type on a calendar sheet, daddy?

D: We do have such games. Now take the calendar for May:

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Choose a number from column 1 and have it in your mind. Choose another from column 3 and have it also in your mind. Add both the numbers and don't tell me the sum. But there is one condition. The sum should be less than 32 when we use a calendar where the number sequence stops with 31.

S: Yes, I have the sum.

D: Your sum is in column 4.

S: Oh! this is terrific! I took 8 from column 1 and 17 from column 3 and added 8 and 17. I got 25 and 25 is in column 4. How do you find it out, daddy?

Daddy, I choose a number from column 2 and another from column 4. I have struck the sum.

D: Your sum is in column 6.

R: Correct! I choose 16 from column 2 and 4 from column 4. I add 16 and 4 and get 20. And 20 is in the 6th column.

S: Can't we choose the numbers from the same column?

D: Choose two numbers from 4 and find out the sum.

S: I have got the sum.

D: Your sum is in column 1.

S: Correct. I choose 11 and 18. Their sum is 29. It is in column 1.
Oh! I see.
4 + 4 is 8 and 8 is column 1 number.

P: I have a problem. I choose 22 from column 1 and 10 from column 3. I add 22 and 10 and get 32. But 32 is not seen in the calendar.

D: In such cases you imagine that numbers do not stop with 31 and continue beyond 31.

P: In that case my sum is in column 4. Shall I tell you the secret? Add the top numbers of the columns chosen and their sum gives the top number of the column that has the sum of the chosen numbers.

D: Oh, that is superb!

P: I have another problem. I choose 11 from column 4 and 20 from column 6. Adding the chosen numbers, I get 11 + 20 = 31. And 4 + 6 = 10, but there is no 10th column.

R: But 31 is in column 3.

S: 10 is in column 3.

P: I can fix up the column having 10. But how to get 3 without searching for 10?

D: Numbers in each column are related to its top number. Can you find it?

R: Numbers in each column are in additive sequence. Each number is greater than the previous number by 7.

S: The numbers in the last column are multiples of 7

P: Oh! I see. If we add 1 to each of the multiples of 7, we find the sums in the 1st column.

D: We can also say that numbers in any column divided by 7 leave the same remainder and that remainder is the top number of that column.

S: But how can we divide 4 by 7?

R: I know it. 7 goes into 4 zero times and the remainder is 4

$$
\begin{array}{r}
0 \\
7\overline{)\,4} \\
0 \\
\hline
4 \\
\hline
\end{array}
$$

P: That is correct.

R: So the key numbers in this column sum game are 1, 2, 3, 4, 5, 6, 7. Am I right?

S: 7 divides 7 and the remainder is 0.

D: Write numbers from 0 onwards in 7 columns and see what happens.

S:

0	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27

and so on. The last column of calendar arrangement appears in the first column and every other column is pushed to its next column.

D: We can therefore call the numbers in the first column as remainder 0 or R_0 numbers, the numbers in the second column as remainder 1 numbers or R_1 numbers and so on. Suppose we write numbers from 0 onwards in 6 columns, what shall we find?

S: Let me write and see

0	1	2	3	4	5
6	7	8	9	10	11
12	13	14	15	16	17
18	19	20	21	22	23

and so on.

R: We get remainder 0 numbers, remainder 1 numbers and so on, until we stop with remainder 5 numbers.

D: What do we divide the numbers by?

S: 6.

D: If we divide the counting numbers by 8, how many kinds of remainder numbers shall we get?

What are they?

R: 8. They are R_0, R_1, R_2, R_3, R_4, R_5, R_6 and

D: *'Leaving the same remainder as'* is a relation and Gauss called it congruence relation. He also gave a beautiful notation for it.
$9 \equiv 23 \pmod 7$, read as 9 congruent to 23 modulo 7.

P: This means 9 and 23 leave the same remainder when divided by 7.

D: Modulo means measure. When you measure out 9 and 23 with 7 as unit, the remainders are the same.

R: So the difference between 9 and 23 is 14 and 2×7 is 14. Can't we say that the difference between 9 and 23 is a multiple of 7?

D: We can also say so. This congruence relation behaves like equality relation.

P: What does it mean?

D: Take the equality relation $a = b$. Is not
$a + c = b + c$?
If $c = d$, is not $a + c = b + d$?
Can't we do similar things with congruence relation?

P: Oh! I get it. Take $9 \equiv 23 \pmod 7$ from column 2. Now $9 + 2 \equiv 23 + 2 \pmod 7$. Correct. That is $10 \equiv 17 \pmod 7$ from column 3. And
$9 + 10 \equiv 23 + 17 \pmod 7$ from column 5.
Oh! this is our column sum game.

R: Can't we have *column difference game?*

D: You can have it. How do you interpret $5 - 3$?

R: 'Subtract 3 from 5'. Also 'add -3 to 5'.

P: Yes, subtracting is nothing but adding the opposite.

D: That means addition takes care of subtraction. There is no need to talk of subtraction separately.

S: Can't we have *column product game:* Choose 9 from column 2 and 3 from column 3. Multiply 9 by 3, you get 27. It should be in column $2 \times 3 = 6$. Yes, it is!.

D: Now take the equality relation $a = b$. Is not $ac = bc$? If $c = d$, is not $ac = bd$?
Can't we have similar things for the congruence relation?

P: I am sure we can. Take
$8 \equiv 15 \pmod 7$ from column 1.
Now $8 \times 2 \equiv 15 \times 2 \pmod 7$. It is correct.
And $2 \equiv 9 \pmod 7$ from column 2
So $8 \times 2 \equiv 15 \times 9 \pmod 7$
That is, $16 \equiv 135 \pmod 7$. Is this correct?

R: 135 - 16 should be a multiple of 7. 119 is 7x17. Yes, it is correct.

P: We can also have for congruence relation
$a^2 \equiv b^2$, $a^3 \equiv b^3$ etc.
In general we can have $a^n \equiv b^n$. Right?

D: You are absolutely right.

P: What about *division?* In other words, what about multiplication by a fraction. Take fractional numbers. If $a = b$,
then $\frac{a}{c} = \frac{b}{c}$. If $c = d$, then $\frac{a}{c} = \frac{b}{d}$. Of course c or d should not be zero. We can also say $\frac{1}{c} \times a = \frac{1}{c} \times b$ and if $\frac{1}{c} = \frac{1}{d}$,
then $\frac{1}{c} \times a = \frac{1}{d} \times b$.

D: In a congruence relation, one has to be very careful. Division is not universal. We shall consider more of this some other time.

9

DAY AND DATE

S: Daddy, often we look up the calendar to know the day for a date. Will these remainder numbers help us in answering such questions?

D: That is a nice question. If we know the day for any date, we can fix the day for a subsequent or an earlier date by using the remainders. As before take the calendar for the month of May 88

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

What day is 3rd?

S: Tuesday,

D: Add 7 to 3.

S: 10

D: What day is 10th?

S: Tuesday

D: So what does 'adding 7' mean?

Not changing the day.

D: O.K. What does 'adding 6' mean?

R: Fixing the previous day.

D: What does 'adding 8' mean?

R: Fixing the next day.

D: What does 'adding 1' mean?

S: Fixing the next day.

P: 'Adding 8' and 'adding 1' fix the next day. Oh! I see. 8 and 1 are R_1 numbers.

R: I see a pattern now: 1 9 17 25. That is formed by keeping on adding 8. So since 1 is Sunday, we get 9 Monday, 17 Tuesday, 25 Wednesday.

S: I also see another pattern: 6 12 18 24 30 This is formed by keeping on adding 6. Since 6 is Friday, we get 12 Thursday, 18 Wednesday, 24 Tuesday, 30 Monday.

P: Going by their remainder numbers, we get 6, 5, 4, 3 and 2. This shows why the days are coming backwards here.

S: It is interesting!

D: What day is 9th?

S: Monday.

D: 11 days hence, what will be the day?

S: 9 + 11 = 20, 20th is Friday.

R: Let us go by their remainder numbers. 9 is remainder 2 number. 11 is remainder 4 number. 9 + 11 is congruent to 2 + 4 = 6 and 6th is Friday.

P: 15th is Sunday. It is R_1 number. 23 days what will be the day?

S: 23 is remainder 2 number. Adding 1 and 2 we get 3. Remainder 3 numbers show it is Tuesday.

R: Let me check up with the calendar in the ordinary way.

15 + 23 = 38

31 + 7 = 38

7th June is Tuesday. So it is correct.

P: That means with the calendar for May we can find the day for any date of the year. Am I right?

D: Not only for this year, but also the day for any date of any year. But it should be Gregorian. Do you remember Julian and Gregorian calendars?

R: Yes, you have told us. I have a question. Why May, daddy? Can't we use the calendar of any month to find the day of any date of the year?

D: That would mean, going backwards sometimes. As we can subtract any remainder number from any other remainder number, what Ravi says can be done. Let us take an example and see.

S: 17th June is Friday. What was the day 11 days before?
17 - 11 = 6 and 6th is Monday. But we should do this by using remainder numbers. 17 is remainder 3 number and 11 is remainder 4 number. Now we should find 3 - 4. 3 and 10 are congruent. So we can find instead 10 - 4. It is 6 and 6th is Monday. Yes. It works.

R: 6th March is Sunday. 40 days before, what was the day?

S: 40 is remainder 5 number. 6 - 5 = 1.

1st March is Tuesday. So 40 days before 6th March is Thursday. But the month is different.

S: Let me check up by going backwards:

						Tues				
6	5	4	3	2	1	29	28	27	26	25
	24	23	22	21	20	19	18	17	16	15
	14	13	12	11	10	9	8	7	6	5
	4	3	2	1	31	30	29	28	27	26
It is correct.					Wed					Thu

D: Alright. What day will be 1.1.89

R: 1.1.88 is Friday. The question is to find the day 366 days hence, as 1988 is a leap year.

D: Wait a minute. $\frac{366}{7}$ = 52 R 2. So 366 is a remainder 2 number and 1 is a remainder number. And 1 + 2 = 3. So 3rd is Sunday.

R: Wait a minute. Let me check up with the calendar. 31.12.88 is Saturday. So 1.1.89 is Sunday. Correct.

S: Now what day was 1.1.87

R: 1.1.88 is Friday. The question is to find the day 365 days before 1.1.88 since 1987 is not a leap year.

S: $\frac{365}{7}$ = 52 R 1.
365 is a remainder 1 number. So 1 - 1 = 0. 0 or 7 is a Thursday.

R: Let us check with the 1987 calendar on the over there. Yes, 1-1-87 is Thursday.

P: Can't we simplify the calendar using the wonderful role of remainder numbers?

D: Yes, we can just have the condensed calendar in a sheet of post card size.

R: Really! That will be exciting! Give us some hints to make one.

D: It should show the maximum number of dates.

S: You mean the dates from 1 to 31.

D: Yes, can you remember the number of days of each month?

R:

Jan	Feb	Mar	Apr	May	June
31	28 or 29	31	30	31	30
July	Aug	Sep	Oct	Nov	Dec
31	31	30	31	30	31

D: Then we should fix the day to show the first date of each month.

R: O.K. I shall use the remainder numbers and fix the days.

S: Then keep the January 88 sheet and give the rest of the sheets of the calendar to me. I shall check up your answers.

R: Jan 1 is Friday.
Feb 1 is 31 days hence, 31 is R_3. So $1 + 3 = 4$ and 4th is Monday. Feb 1 is Monday.

S: Correct!

R: Mar 1 is 29 days hence, 29 is R_1. $4 + 1 = 5$ and 5 is Tuesday. Mar 1 is Tuesday.

S: Correct!

R: April 1 is 31 days hence; 31 is R_3. So $5+3 = 8$. 8 is the same as 1 and 1 is Friday. So April 1 is Friday.

S: Correct.

R: Let me work out the rest and tell you.

May 1 is Sunday.
June 1 is Wednesday
July 1 is Friday
Aug 1 is Monday
Sep 1 is Thursday
Oct 1 is Saturday
Nov 1 is Tuesday
Dec 1 is Thursday

S: All correct.

P: Now January, April and July start on Friday, October on Saturday, May on Sunday, February and August on Monday, March and November on Tuesday, June on Wednesday, September and December on Thursday.

D: We can put them down thus:

1, 4, 7	Fri
10	Sat
5	Sun
2, 8	Mon
3, 11	Tue
6	Wed
9, 12	Thur

Just as you have fixed the days by referring to January alone, we can retain the calendar fo January and show in condensed form the calenda of other months. Here is the condensed calendar for the year 1988. This is one way.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
	31					1	2
	3	4	5	6	7	8	9
	10	11	12	13	14	15	16
	17	18	19	20	21	22	23
	24	25	26	27	28	29	30
Months							
1, 4, 7	3	4	5	6	7	1	2
10	4	5	6	7	1	2	3
5	5	6	7	1	2	3	4
2, 8	6	7	1	2	3	4	5
3, 11	7	1	2	3	4	5	6
6	1	2	3	4	5	6	7
9, 12	2	3	4	5	6	7	1

S: Let me choose a date 21.9.88. How shall we use this condensed calendar to find the day for this date?

P: The day corresponding to 21 across the 9th month is 6 and the 6th is Wednesday.

R: Give me a date. I shall read the day.

S: 24. 11. 88.

R: The day corresponding to 24 across 11th month is 7 and 7th is Thursday.

S: Correct!

P: Can't we simplify the calendar further? I have an idea. We shall first work out what numbers are to be added to remainder numbers of other months to get the January remainder numbers.

For 1, 4, 7 it is 0 or + 7

For 10 it is + 1

For 5 it is + 2

For 2, 8 it is + 3

For 3, 11 it is + 4

For 6 it is + 5

For 9, 12 it is + 6

R: So what?

P: As before we shall take the January calendar and in the bottom portion put down the months in the order of increase in remainders to go with the remainders of January. Here it is:

Jan '88

Sun	Mon	Tue	Wed	Thu	Fri	Sat
31				0	1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
2 8	3 11	6	9 12	1 4 7	10	5

R: Give that to me. Tell me a date. I shall fix the day.

P: Well, 12.3.88.

R: $12 \equiv 5$, $3 \to 4$ and $5 + 4 = 9$ and $9 \equiv 2$. 2 is Saturday. So, 12.3.88 is Saturday. Am I right?

P: You have got it!

S: Come on. Let me have my chance.

R: What day is 19.8.1988 ?

S: $19 \equiv 5$, $8 \to 3$, $5 + 3 = 8$, $8 \equiv 1$
1 is Friday. So 19.8.1988 is Friday.

P: I think I can condense the calendar still further. Why should it show all the dates?
What we want is the remainder for a date. And we get it by subtracting as many multiples of 7 as we can from the date.

R: I see your point. I shall do it for you. It can be done on a book mark.

Jan '88

Sun	Mon	Tue	Wed	Thu	Fri	Sat
3	4	5	6	7 or 0	1	2
2	3	6	9	1 4	10	5
8	11		12	7		

S: Oh! this is wonderful. Every year our daddy can print such condensed calendars and we can give these as gifts to our friends.

D: We have seen that the set of remainders 0, 1, 2, 3, 4, 5, 6 play a prominent role in fixing the days and in locating column sum, column difference and column product. So we shall study their behaviour separately. We shall first make their addition table.

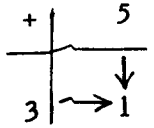
R: I shall fix the table.

S: You want the calendar?

R: No. I don't need it. Here is the addition table:

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

S: This table gives strange addition facts



R: If you take 3 and 5 as ordinary numbers, then it is not 1. But if you take 3 and 5 as remainders numbers, remainders got on division by 7, then it

is correct.

S: Oh! I see. A remainder 3 number + a remainder 5 number is a remainder 1 number..

R: Can you give an example?

S: I have to add 3 and 5 to multiples of 7 to get respectively a remainder 3 number and a remainder 5 number.

$14 + 3 = 17$, $21 + 5 = 26$. Now

$17 + 26 = 43$ and $\frac{43}{7} = 6$ R 1; so it is correct.

P: Is not 0 a multiple of 7? So we can simply add 3 to 0 and 5 to 0 and get 3 and 5 as required remainder numbers.. Adding them, you get 8 and 8 is a remainder 1 number.

S: Now I notice it. Thank you.

D: There is something more interesting about this. What should be added to 3 to get 0 in the set of whole numbers?

R: There is no such whole number.

D: But in the set of integers?

R: I know it. It is -3.

D: But here, in the set of remainders?

R: 3 plus what is zero? The table shows that it is 4.

P: Oh, that means the opposites of these numbers are contained in the set itself!

Is that not wonderful? Except 0, no whole number has a whole number as its opposite. That is why integers are created. But in a set of remainders for some modulo (≥ 2) the opposites of remainders are got from the remainders themselves. List the remainders and write below each its opposite.

R: I will do it:

Remainders	0	1	2	3	4	5	6
Opposites	0	6	5	4	3	2	1

Oh, except zero, these are 7 complements.

D: Take the squares of numbers 0 to 6 and find their remainders for modulo 7

R: I shall take them in complementary pairs for 7.

$1^2 \equiv 1 \pmod 7$

$6^2 \equiv 1 \pmod 7$

$2^2 \equiv 4 \pmod 7$

$5^2 \equiv 4 \pmod 7$

$3^2 \equiv 2 \pmod 7$

$4^2 \equiv 2 \pmod 7$

It is amazing they stay together.

P: But only 1, 2 and 4 are got as remainders.

D: 1, 2 and 4 are called quadratic residues.
3, 5 and 6 are called quadratic non-residues.
Gauss was fascinated by this.

P: So for each modulo, some will be quadratic residues and some quadratic non-residues.

D: Yes, it is so. Let us now compare subtracting

the set of whole numbers and in the set of remainders (modulo 7). What is 2 - 5 in the set of whole numbers?

R: There is no answer. Only in the set of integers, 2 - 5 is -3.

D: Now what is 2 - 5 in the set of remainders (modulo 7)?

P: What should be added to 5 to get 2? The table shows it is 4.

+	5
4	→ 2

So subtraction of any remainder number from any remainder number in the set can always be done and we get one of the remainder numbers as the answer.

D: There is another surprising thing about these remainder numbers.
If 2, 4 and 6 are ordinary numbers,
6 - 4 = 2, 4 - 2 = 2. We know that
6 > 4 > 2. But here 2, 4, 6 are remainder numbers. Let us look at the addition table modulo 7 and find relations among 2, 4 and 6.

R: We have 6 - 4 = 2, 6 - 2 = 4

P: Not only these; we also have 4 - 6 = 5
and 2 - 6 = 3
Oh! This is disturbing! For these show that 6 > 4 and 4 > 6, 6 > 2 and 2 > 6 which is absurd.

D: Yes, we have to accept that we cannot arrange the remainder numbers in ascending or descending order. That is to say the remainder numbers or residues have *no order property*. Now we shall study their behaviour under multiplication.

R: I shall fix the multiplication table.

You need the calendar now?

R: I can fix the table without it. But to make it fast, let me have it.

x	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

The table is ready.

S: Let me check. A remainder 6 number multiplied by a remainder 4 number should be a remainder 3 number.
A remainder 6 number is 13.
A remainder 4 number is 11.
Multiplying them, I get 143;

$$\frac{143}{7} = 20 \text{ R } 3$$

Yes, 143 is a remainder 3 number.

D: In the system of whole numbers, you cannot find a whole number to multiply to get 5. That is $3 \times \square = 5$ has no solution. But in the set of remainders (modulo 7) there is answer. What is it?

R: The table shows it is 4 for 3 x 4 = 5.

P: Oh! I see what it means. There is no need here, for introducing fractions and then consider the whole numbers as fractions with 1 in the denominator in their reduced or simplest form. Except for division by 0 as in the set of whole numbers, you can divide one remainder number by another and find answer.

D: You know rational numbers, that is numbers formed by integers divided by non-zero integers. There is no first rational number and there is no last rational numbers and the set of rational numbers is an infinite set. The fascinating thing here is that a finite set of remainders (modulo 7) behaves like an infinite set in the sense that you can perform .....

R: Addition, subtraction, multiplication and division without remainder.

P: It is simply amazing! What is the name given to a set in which one can do the four operations?

D: There is a crude way of describing such sets. Anyhow, in mathematicians' language, such a set is called a *field.*

P: Daddy, I have a question. What would happen when we change to smaller modulo? Won't we be getting smaller sets?

D: Yes, there can be the smallest set.

P: It should consist of 0 and 1 for modulo 2.

S: What if we write numbers in 6 columns?

D: Write and see. Start from 0.

S: 0 to 5 will be the six column heads. O.K. Here it is:

0	1	2	3	4	5
6	7	8	9	10	11
12	13	14	15	16	17
18	19	20	21	22	23
24	25	26	27	28	29
30	31	32	33	34	35

and so on.

D: Good! What are the remainder numbers?

S: 0, 1, 2, 3, 4 and 5.

D: Look what has happened. If I give you a number, can you show me in which column it has its place?

R: Give me a number.

D: 113.

R: $\frac{113}{6} = 18 \text{ R } 5.$

P: That is $113 \equiv 5 \pmod 6$

S: 113 is in the last column headed by 5.

D: So every number has its place in any one of the columns. Right?

R: Yes.

D: Can a number appear in more than one column?

S: No.

P: There are six remainder sets. No two of them have a common number.

D: That is so. Such a scheme in mathematician's language is called *partitioning* and the subsets turn out to be *disjoint.*

P: I get it. Just by writing numbers in a certain number of columns, wonderful things are seen to happen:

(1) Modulo gets fixed and it is the number of columns.
(2) A finite set of remainders for that modulo is obtained.
(3) The set of whole numbers gets partitioned into disjoint subsets.

R: Why? We can also play column sum game, column difference game, and column product game.

D: In the set of remainders modulo 7, or if you like in *7-arithmetic*, we have seen you can do addition, subtraction, multiplication and division without remainder. In other words, the set of remainders modulo 7 is a field. We shall see whether it is so for any other modulo. Take the set of remainders modulo 6.

R: The remainders are 0, 1, 2, 3, 4 and 5. We should frame the tables of addition and multiplication. Shouldn't we?

S: I shall fix the addition table for this *6-arithmetic*.

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

R: This is just like modulo 7 addition table.

P: So we can do addition and subtraction and get answer always.

S: Now I shall fix the multiplication for the *6-arithmetic*.

x	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

R: Oh, God! This is different from multiplication table for 7-arithmetic

D: What you observe is correct. Can you spot out some of the differences?

P: 3 x 2 = 0, 3 x 4 = 0. We don't have such products in modulo 7 table. We don't have such products in the set of whole numbers, in the set of fractional numbers, in the set of integers, in the set of rational numbers etc. If two non-zero numbers are multiplied, the product is never zero in these sets. In other words, zero has no divisors in these sets.

D: O.K. Take this modulo 6 multiplication table. Can you solve 2 x $\square$ = 4?

R: There is more than one answer. 2 x 2 = 4 and 2 x 5 = 4. So 2 or 5 can go into the frame.

D: Can you find 3/4? That is to say, can you solve 4 x $\square$ = 3.

P: Oh, I see. There is no solution.
4 x 0 = 0, 4 x 1 = 4, 4 x 2 = 2,
4 x 3 = 0, 4 x 4 = 4, and 4 x 5 = ? and we don't at all get 3 as product.

R: So we cannot do division always in the set

remainders modulo 6.

D: You are smart! So in the set of remainder numbers modulo 6, we can do addition, subtraction. and multiplication always but not division.

P: So this cannot be a field.

D: Yes, you are right. It is called a *ring*.

P: Now the question is how to find out when the set of remainders for a certain modulo is a field or a ring.

D: That is a good question.

S: Why? We can form addition tables and multiplication tables and examine them.

R: I think multiplication table will be just enough.

P: Of course. But that would still be tedius.

R: I think I have an answer. When we have an odd modulo, it will be a field. When we have an even modulo, it will be a ring.

D: It is not that simple! Build addition table and muliplicatiton table for modulo 4, for modulo 5. and for modulo 9 and see.

P: Why should we build both the tables for each modulo? Will it not be enough if we build multiplication tables alone?

D: Well, try and see.

R: First I will build modulo 9 multiplication table.

S: I will, in the mean time, build modulo 4 and modulo 5 multiplication tables.

R:

x_9	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8
2	0	2	4	6	8	1	3	5	7
3	0	3	6	0	3	6	0	3	6
4	0	4	8	3	7	2	6	1	5
5	0	5	1	6	2	7	3	8	4
6	0	6	3	0	6	3	0	6	3
7	0	7	5	3	1	8	6	4	2
8	0	8	7	6	5	4	3	2	1

S:

x_4	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

x_5	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

R: Modulo 9 multiplication table is like modulo 6 multiplication table. So what I first guessed is not correct.

S: But modulo 5 multiplication table is like modulo 7 multiplication table.

P: So modulo 4, modulo 6. and modulo 9 multiplication tables form a pattern and modulo 5, modulo 7 multiplication tables form another pattern.
Oh! Yes. I discover the pattern. 4, 6 and 9 in

composite numbers, numbers having more than 2 factors. 5 and 7 are prime numbers. numbers having only two factors. *For the operations of addition and multiplication the set of remainders for a prime number modulo is a field and the set of remainders for a composite number modulo is a ring.*

D: That is a great discovery! One more interesting point. Write whole numbers in a sequence. Write remainders for modulo 3 below it in a sequence.

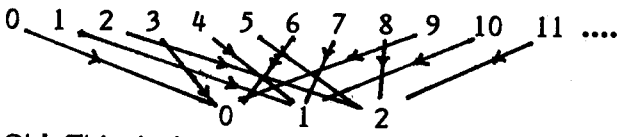
R: Here they are:

0 1 2 3 4 5 6 7 8 9 10 ...

0 1 2

D: Connect them by the relation 'leaving the same remainder as'

S: I will do it:



Oh! This is beautiful!

D: In mathematicians' language, the whole numbers can be said to form a *domain* and the remainders *codomain* and the connecting arrows show the mapping.

P: 0, 3, 6, 9 ..... are mapped on 0;
1, 4, 7, 10 ..... are mapped on 1;
2, 5, 8, 11 ..... are mapped on 2.

D: What kind of mapping is this?

P: More than one whole number is mapped on each remainder. All the remainders are used. So it is *many - one, onto mapping.* It is not an ordinary relation. It is a functional relation if we recall what you introduced to us last year. Right?

D: Good! Now we can consider 0, 1, 2 as images of whole numbers. What are the images of 5 and 10?

R: The images are 2 and 1.

D: What is the image of the sum (5 + 10) or 15?

R: 0

S: Zero is 2 + 1

P: That is, zero is the sum of the images 2 and 1. And so on. So the *image of the sum is the same* as *the sum of the images.*

R: Will there be a similar thing for products? Let us try. Images of 5 and 10 are 2 and 1. Image of the product 5 x 10 or 50 is 2 and product of the images 2 and 1 is also 2. And so on.

P: So we find the *image of the product is the same as the product of the images.*

D: This structural property is termed *homomorphism* by mathematicians. Well, a beautiful divisibility property of numbers can be got from a prime number modulo multiplication table.

P: What is that property?

D: Take for instance modulo 5 multiplication table, omitting 0 column and 0 row.

S: Here is it:

$\times_5$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

Oh! We are getting a Latin square in 1,2,3 and 4

D: Now $3 \times 1 = 3$, $3 \times 2 = 1$, $3 \times 3 = 4$ and $3 \times 4 = 2$ under modulo 5.
Multiply these together.

R: $3 \times 1 \times 3 \times 2 \times 3 \times 3 \times 3 \times 4 = 3 \times 1 \times 4 \times 2$ for modulo 5.

P: We can write this as
$3^4 \times 1 \times 2 \times 3 \times 4 = 1 \times 2 \times 3 \times 4$ (module 5)

D: Can't you simplify it?

P: Oh! It is a prime number modulo. We can divide both sides by $1 \times 2 \times 3 \times 4$ and $3^4 \equiv 1$ (modulo 5)

D: So what is $3^4 - 1$?

R: $3^4 - 1 \equiv 0$ (modulo 5)

D: What does that mean?

P: $3^4 - 1$ is divisible by 5

D: If we use other rows what do we get?

R: $1^4 - 1$ is divisible by 5;
$2^4 - 1$ is divisible by 5;
$4^4 - 1$ is divisible by 5.

D: If we take the multiplication table for prime modulo 7, what similar facts can we find?

$1^6 - 1$ is divisible by 7;

$2^6 - 1$ is divisible by 7;

$3^6 - 1$ is divisible by 7;

$4^6 - 1$ is divisible by 7;

$5^6 - 1$ is divisible by 7;

$6^6 - 1$ is divisible by 7.

But $7^6 - 1$ is not divisible by 7. What about $8^6 - 1$? Is it divisible by 7?

D: Can't you use the congruence relation in solving this?

P: Let me see. What I have to show is $8^6 \equiv 1$ (mod7). Right?

D: Go ahead.

P: $8 \equiv 1 \pmod 7$,
so $8^6 \equiv 1^6 \pmod 7$.
That is, $8^6 \equiv 1 \pmod 7$, Right?

D: What about $9^6 - 1$?

P: We should find
if $9^6 - 1 \equiv 0 \pmod 7$ is true
O.K. $9 \equiv 2 \pmod 7$;
So $9^6 \equiv 2^6 \pmod 7$.
Since $2^6 - 1$ is divisible by 7,
$9^6 - 1$ is also divisible by 7.

D: So what is the grand number property we are getting?

P: Choose a prime p, say.
Choose any number, say, n which is not a multiple of p. Then $n^{p-1} - 1$ is divisible by p. What a fine discovery! What a deep result! Who found this first?

D: Fermat, the great French mathematician of the 17th century.
Well, I will be away for a fortnight. I would be happy if you could investigate in the mean time how to find the day for any date in any year.

P: In the Gregorian Calendar?

D: Yes, that is it; we are using only that calendar now.

R: We will try our best to find it out before you return. Daddy, I have a question.

S: How many kinds of arithmetic are there, daddy?

D: Now from what you have seen we can no more say arithmetic. We have to say *arithmetics*. You can have as many arithmetics as you like.

P: But they will have only two structures, field and ring.

D: Yes, you are correct.

S: Thank you, daddy?

R: We really had an exciting time, daddy, with this calendar.

P: Oh, God! So much from a calendar!! We are grateful to you, daddy.

been set up by the Extension Services Department of St. Christopher's College of Education, Madras and its Annual Inter School Mathematics Project Exposition is held. He brought out in 1968, two volume Ramanujan Memorial Number containing a lot of newly unearthed material about Ramanujan. Mr. Srinivasan is the author of numerous text books, supplementary and enrichment books in mathematics in English and Tamil. His articles in Mathematics and Mathematics education have appeared in *Mathematics Teacher* (India), *Mathematics Teacher* (USA) *Studies in Mathematics Education* (UNESCO). Popular articles in mathematics and mathematics education have been appearing in leading national newspapers : *Hindu* and *The Dinamani*.

He is a believer in and crusader for conference culture among teachers. As a full member of the International Congress on Mathematics Education he has been participating in its four yearly meets since 1972, Exeter (UK), Karlsruhe (FRG), California (USA), Adelaide (Australia), Budapest (Hungary).

He has been advocating *Introductory Algebra for 8 years olds*. He visited Denmark, Sweden, Norway and Finland in Aug. '88 and successfully advocated this revolutionary move in mathematics education.

He is a founder life member and the present Academic Secretary of the Association of Mathematics Teachers of India. He has authored its *Teachers' Instructional Guides on Introduction to Creativity of Ramanujan* To mark Ramanujan Centenary. He is a member of the Curriculam Devalopment Centre set up by the U. G. C. at the Ramanujan Institute of Advanced Study in Mathematics, Madras.